Correction Algorithm of Doppler Effect on the Comb-Based FSO-TWTFT for Satellite Application

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***Summary*—We further analyze the Doppler effect caused by high-velocity platform motion for the comb-based free-space optical two-way time-frequency transfer technique. A novel synchronization algorithm is proposed to correct the Doppler-induced error.**

***Keywords*—optical frequency comb transfer; satellite motion; Doppler effect**

I. Introduction

The comb-based free-space optical two-way time-frequency transfer (FSO-TWTFT) technique has become a powerful candidate for establishing the future satellite optical clock network[1-5]. However, for the satellite application, the Doppler effect caused by high velocity brings significant challenges to the comb-based FSO-TWTFT.

In ref. [4], the Doppler effects on the comb center frequency and repetition frequency are both considered, which induce a synchronization error by the delay-Doppler coupling and the asynchronous sampling, respectively. Here, we focus on the Doppler effect on the comb repetition frequency. In ref. [4], the asynchronous sampling can be seen as the influence of Doppler effect on the bidirectionally transferred comb pulses. Due to the continuous platform motion, the asynchronous transmission of comb pulses will result in a nonreciprocal two-way link delay, which can induce a 20 ps error in the clock difference calculation under the velocity V=25 m/s. After adding the corresponding correction term, the time deviation between two sites can be synchronized under 1 fs. However, for the satellite application with high velocity up to 6 km/s, the Doppler effect can cause a significant variation of the comb repetition frequency, and then the transferred comb pulses are no longer equivalent to the original comb pulses. In this case, a synchronization error can also be generated. Moreover, the acceleration of satellite also needs to be considered. For instance, when setting the repetition frequency difference of the bidirectional combs at 1 kHz, the nonreciprocal link delay caused by 20 m/s^2 acceleration can reach 15 fs, degenerating the sub-fs precision that can be achieved by the comb-based FSO-TWTFT.

In this article, the synchronization error caused by the Doppler effect on the high-velocity platform is further investigated. A novel signal processing method is proposed by determining the phase relationship of the transferred comb pulses and the original comb pluses, and the correction related to the acceleration is taken into consideration.

II. Theory

Figure 1 (a) shows the schematic diagram of the comb-based FSO-TWTFT for synchronizing the local site and remote site. Both sites can be a mobile platform. The comb pulses outputted by comb A and comb B are used as the clock ticks and have the same repetition frequency (denoted by f_r). A comb X with repetition frequency $f_\mathrm{r}+\Delta f_\mathrm{r}$ is introduced to accomplish the two-way time comparison and the high-resolution time measurement. The signals from the comb B and comb X propagate bidirectionally along a common FSO link. Here, we define the transferred combs as comb Xt and comb Bt.

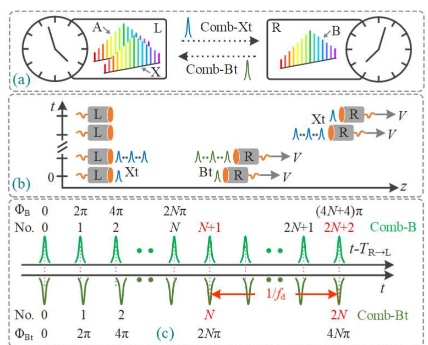

Fig. 1 (a) System architecture of the comb-based FSO-TWTFT. (b) Signal transceiving process. (c) Pulse sequences of the original comb B and the transferred comb Bt.

Figure 1 (b) illustrates the signal transceiving process of the comb-based FSO-TWTFT, in which the remote site keeps away from the local site at a velocity V. For the comb Bt, due to the platform motion, the pulse propagation delay increases with the increasing of the launching time, and the increased delay between two adjacent pulses is equal to $V/(cf_\mathrm{r})$. Thus, the pulse repetition period of the comb Bt received at the local site becomes $T_\mathrm{Bt}=1/f_\mathrm{r}+V/(cf_\mathrm{r})$, which differs from the f_r of original comb B. In frequency domain, this phenomenon can be seen that a Doppler frequency shift $f_\mathrm{d}=Vf_\mathrm{r}/c$ is generated. Similarly, the pulse repetition period of the comb Xt received at the remote site becomes $T_\mathrm{Xt}=1/(f_\mathrm{r}+\Delta f_\mathrm{r})+V/[c(f_\mathrm{r}+\Delta f_\mathrm{r})]$.

Figure 1 (c) presents the pulse sequences of the original comb B at the remote site and the comb Bt received at the local site. Due to the Doppler effect, the pulse repetition period of comb Bt becomes longer than that of the comb B, and after a duration of $1/f_{\mathrm{d}}$, the pulse interval of comb B and comb Bt with the same ordinal number will become higher than the pulse period $1/f_{\mathrm{r}}$. This phenomenon will affect the process of solving periodic waveform ambiguities in [4], which directly set the phase of comb B being equal to that of the comb Bt. A time error of $1/f_{\mathrm{r}}$ can be induced especially when $f_{\mathrm{d}} > \Delta f_{\mathrm{r}}/2$. For the satellite applications, the f_{d} caused by high-speed satellite motion can easily reach this condition.

To correct the Doppler-induced error, the phase relationship between the original combs and the transferred combs needs to be established. For conveniently comparing the theory in this report and that in [4], we use the same notation, apart from that the transferred combs is written as comb Xt and comb Bt. Thus, according to the phase variation shown in Fig. 1 (c), we can link the phase of the transferred combs ($\Phi_{\mathrm{Bt}}(t, z_{\mathrm{A}})$ and $\Phi_{\mathrm{Xt}}(t, z_{\mathrm{B}})$) with that of the original combs ($\Phi_{\mathrm{B}}(t, z_{\mathrm{B}})$ and $\Phi_{\mathrm{X}}(t, z_{\mathrm{A}})$) by

$$\begin{aligned} \Phi_{\mathrm{B}}(t-T_{\mathrm{BA}}, z_{\mathrm{B}}) &= \left[1+V/c+a\cdot\Phi_{\mathrm{Bt}}(t,z_{\mathrm{A}})/(4\pi c f_{\mathrm{r}})\right]\Phi_{\mathrm{Bt}}(t,z_{\mathrm{A}}) \\ \Phi_{\mathrm{X}}(t-T_{\mathrm{AB}}, z_{\mathrm{A}}) &= \left[1+V/c+a\cdot\Phi_{\mathrm{Xt}}(t,z_{\mathrm{B}})/(4\pi c f_{\mathrm{r}})\right]\Phi_{\mathrm{Xt}}(t,z_{\mathrm{B}}) \end{aligned} \tag{1}$$

where a is the acceleration. It is worth emphasizing that after linking the phase relationship between the original combs and the transferred combs, not only the error $1/f_{\mathrm{r}}$ discussed above can be eliminated, but also the asynchronous sampling considered in [4] can be removed automatically, not needing to add a correction term separately.

When demodulating the clock difference Δt_{AB}, the phase and phase difference of combs at the LOS interferogram peaks need to be determined, which satisfy $\Phi_{\mathrm{X}}(t_{p\mathrm{AX}}, z_{\mathrm{A}}) - \Phi_{\mathrm{A}}(t_{p\mathrm{AX}}, z_{\mathrm{A}}) = 2\pi p_{\mathrm{AX}}$, $\Phi_{\mathrm{X}}(t_{p\mathrm{BX}}, z_{\mathrm{A}}) - \Phi_{\mathrm{Bt}}(t_{p\mathrm{BX}}, z_{\mathrm{A}}) = 2\pi p_{\mathrm{BX}}$, $\Phi_{\mathrm{Xt}}(t_{p\mathrm{XB}}, z_{\mathrm{B}}) - \Phi_{\mathrm{B}}(t_{p\mathrm{XB}}, z_{\mathrm{B}}) = 2\pi p_{\mathrm{XB}}$, $\Phi_{\mathrm{A}}(t_{p\mathrm{AX}}, z_{\mathrm{A}}) = 2\pi k_{\mathrm{pAX}}$, $\Phi_{\mathrm{A}}(t_{p\mathrm{BX}}, z_{\mathrm{A}}) = 2\pi k_{\mathrm{pBX}}$, and $\Phi_{\mathrm{B}}(t_{p\mathrm{XB}}, z_{\mathrm{B}}) = 2\pi k_{\mathrm{pXB}}$, respectively. Here, the determination of number k_{pAX}, k_{pBX}, and k_{pXB} can be easily realized by pulse counting. Moreover, the p_{AX} can also be easily obtained because the comb A and comb X are both located in the local site. However, the determination of p_{BX} and p_{XB} needs the cooperation of coarse clock difference measurement with precision higher than $1/f_{\mathrm{r}}$. Based on Eq. (1), the p_{BX} can be calculated by $p_{\mathrm{BX}} = round\{[\Phi_{\mathrm{X}}(t_{p\mathrm{BX}}, z_{\mathrm{A}}) - (1 + V/c + a\Phi_{\mathrm{Bt}}(t_{p\mathrm{BX}}, z_{\mathrm{A}})/(4\pi f_{\mathrm{r}}))\Phi_{\mathrm{Bt}}(t_{p\mathrm{BX}}, z_{\mathrm{A}})]/4\pi\}$ where $\widetilde{\Phi}_{\mathrm{B}}(t_{\mathrm{pBX}}. z_{\mathrm{B}})$ represents the phase obtained by the coarse clock difference measurement. With the same treatment, the term p_{XB} can also be obtained.

Then, following the same procedure in [4] but link the phase of the transferred combs and the original combs based on Eq. (1), we can get the clock difference finally given by

$$\begin{aligned} \Delta t_{\mathrm{AB}} = \Delta t_{\mathrm{cal}} &+ \frac{\left[1+V/c+a\left(p_{\mathrm{XB}}+k_{\mathrm{pXB}}\right)/(2cf_{\mathrm{r}})\right]\left(p_{\mathrm{XB}}+k_{\mathrm{pXB}}\right)}{f_{\mathrm{r}}+\Delta f_{\mathrm{r}}} \\ &-\left(p_{\mathrm{AX}}+k_{\mathrm{pXB}}-k_{\mathrm{pBX}}\right)/f_{\mathrm{r}}+\Delta f_{\mathrm{r}}\left(p_{\mathrm{AX}}+k_{\mathrm{pAX}}\right)/\left[f_{\mathrm{r}}\left(f_{\mathrm{r}}+\Delta f_{\mathrm{r}}\right)\right] \\ &+\left[1+V/c+a\left(\Delta f_{\mathrm{r}}\left(k_{\mathrm{pBX}}-k_{\mathrm{pAX}}\right)/f_{\mathrm{r}}+k_{\mathrm{pBX}}+p_{\mathrm{AX}}-p_{\mathrm{XB}}\right)/\left(2cf_{\mathrm{r}}^{2}\right)\right] \\ &\times\left[\Delta f_{\mathrm{r}}\left(k_{\mathrm{pBX}}-k_{\mathrm{pAX}}\right)/f_{\mathrm{r}}+k_{\mathrm{pBX}}+p_{\mathrm{AX}}-p_{\mathrm{XB}}\right] \end{aligned} \tag{2}$$

where Δt_{cal} represents the calibration term.

III. RESULTS

Simulations are conducted to analyze the error of the clock difference calculation caused by the Doppler effect on the comb repetition frequency and verify the derived synchronization equation. The value of f_{r} and Δf_{r} is 200 MHz and 2 kHz, respectively, and the acceleration is set at 20 m/s^2. Figure 2 shows the error of Δt_{AB} under different relative velocity V by the method with and without Doppler correction. For the method without Doppler correction, the error is about 50 fs when V=60 m/s, and it will be over 30 ps when V>500 m/s. The red lines in Fig. 2 are the error of Δt_{AB} with Doppler correction, which shows that the error caused by the Doppler effect on the comb repetition frequency can be effectively eliminated. The relative radial velocities between the ground station and the satellite at different orbits and different zenith angles are shown in the inset of Fig. 2. Combining the error of Δt_{AB}, it can be inferred that the comb-based O-TWTFT without Doppler correction is only suitable for the orbit near the GEO satellites. However, the proposed synchronization equation with Doppler correction enables the technology to be applicable for the high-velocity LEO and MEO satellites besides the GEO.

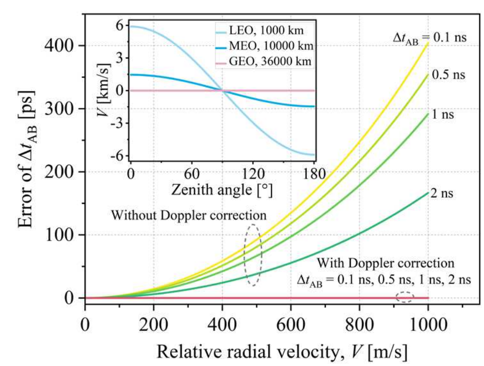

Fig. 2 Error of Δt_{AB} against V with and without Doppler correction. Inset is the relative radial velocity V between the ground station and the satellite.

IV. CONCLUSIONS

In conclusion, from the point of Doppler effect, we further investigate the potential synchronization error caused by the Doppler effect on the high-velocity platform. A solution is proposed by determining the phase relationship of the transferred comb pulses and the original comb pluses, and the correction related to the acceleration is conducted.

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